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THE PROBLEM OF SPACE IN JEWISH MEDIAEVAL PHILOSOPHY

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CHAPTER III

INFINITE SPACE.

ONE of the problems that have troubled the human mind is the problem of space; and one of the aspects of space that have troubled the human mind most, is its infinity. From the philosopher of Stagira to the philosopher of Königsberg, the subject of the infinity of space did not cease to defy and baffle human ingenuity. Our present-day thinkers are mostly silent on this topic. They dread the contest, but they have not overcome it. It still lies like an invincible brute ready to enter the arena. Such being the case, it would be simply preposterous to claim that Jewish philosophy may boast of having solved altogether this overwhelming difficulty, but I do claim that in the course of the progress of Jewish thought some suggestions were made that might lead to a new and better understanding of the problem; and to understand it would be half way to its complete solution.

Let us first turn to Aristotle, who may always serve as a text in any discourse on mediaeval philosophy. His ideas about infinity which are found in the third book of the *Physics*, and in the tenth of the *Metaphysics*, are briefly thus. On the one hand we find that infinity is undeniable.

Time is unbegotten and indestructible. We cannot conceive of a moment of time, a Now which is an absolute beginning of a series of duration. Every Now *looks* on one side to a past and on the other to a future: it has a before and after.¹¹⁴ On the surface it may seem strange that a similar argument could not be advanced to prove the infinity of space: every Here is on one side in touch with a before, and on the other with a beyond. But the argument is really a deeper one. It is repugnant to the entire Aristotelian standpoint of causation, the denial of miraculous creationism, to assume a Now which was not caused by a previous one. Time which marks the duration of the beginningless and endless development of things must in itself be infinite. On the other hand, there must be a limit to material existence. Matter is limited by superficies, and hence finite; and to speak of an infinite number of material bodies is also absurd, for a number is that which can be counted, and hence likewise finite. Besides, an infinite body would be either simple or composite. It could not be, however, a simple body, similar to the one assumed by the earlier physicists, for then it would have consumed by its infinite power all other finite elements, and would have created all things single-handed; but such a monistic theory is contradicted by the fundamental phenomenon of change which implies the existence of contraries in the universe. Nor could that infinite body be a composite without being either a finite number of infinities or an infinite number of finitudes, either alternatives being impossible. Thus after a series of arguments Aristotle concludes the finitude of spatial existence. How then is it—the question is—that infinity seems to be real in time but unreal in space?

¹¹⁴ Comp. *Or Adonai*, p. 62a; also מַפְעֵלוֹת אֱלֹהִים, V, 3.

An explanation for this antinomy Aristotle finds in the nature of the concept. It is in accord with his general dynamic standpoint. Infinity denotes duration rather than simultaneity, succession rather than co-extension. Infinity never *is*, but is perpetually becoming. Hence time can be represented as endless, for it is a succession of fleeting moments, each one vanishing and making room for another. But when you seek to attain the infinite by means of a synthesis of spatial parts, you are aiming not at an endless *process of becoming*, but at an endless *state of being* which is not postulated by the true notion of the infinite. The unlimited is not actual but potential, meaning by the latter term not the potentiality of the brass that can become an accomplished fact in the form of the statue, but a peculiar potentiality like that of time, which though actual only in an insignificant and vanishing moment, constantly unfolds itself in a never-ending succession of decay and regeneration. It is a *process*, not a *state*. The usual meaning of the infinite, says Aristotle, is that beyond which there is nothing, but the true meaning is that which always has something beyond.

This analysis of infinity is extremely suggestive. It might be shown what a host of perplexing difficulties would vanish in this new light, as we shall see in the sequel. But it is unfortunate that Aristotle himself did not fully realize the immense fruitfulness of its suggestiveness. He seemingly forgets very soon this well-defined position, namely, that things are always and everywhere finite, but reveal the infinite in the process of change and duration, just as in the arithmetical convergent series every term is limited and gives us a limited quantity when added up with the preceding terms, but there is the infinity of

progression, a possibility of enlarging the number of one unit to all eternities. For with this distinction between state and process clearly in his consciousness, how could he possibly speak of a realizable infinitesimal by means of division? My impression is that Aristotle fell a victim to his terminology, to his use of 'potentiality', which always implies something actual, to express his notion of infinity,—an expression which, as he himself felt, hardly suits the meaning. The whole distinction between infinite divisibility and infinite augmentation, the former being affirmed and the latter denied, is unintelligible: *practically* no one would believe that we may divide an object *ad infinitum*, and *theoretically*, even the celestial firmament can form no limit to our augmentation. In the history of the Jewish conception of infinity, this latter potential notion was at first dominating until the former progressive notion was taken up and modified by Gersonides. Let us follow closely this meandering path of the idea of infinity through Jewish philosophy.

Beginning with Saadya, we find that the material universe is held to be limited, having a terrestrial centre and a celestial circumference.¹¹⁵ This finitude of matter means also the finitude of space, for, as we have seen, the void was not posited by the earlier Jewish thinkers. Saadya pays more attention to the theory of temporal infinity maintained by Aristotle, the refutation of which theory, though somewhat beyond the pale of this work, is nevertheless relevant because of its application to spatial infinity. It is ridiculous, he holds, to say that time had no beginning, for then an infinite number of points have already elapsed;

¹¹⁵ *Emunot*, I, p. 56: להם תכלית : שהשמים והארץ כיון שהתברר שיש להם תכלית : בהיות הארץ באמצע ובסובב השמים סביבותיהם.

in other words, this present moment would be the final term of an infinite series, but an infinite series is that which cannot be completed.¹¹⁶ Moreover, every passing day is added to the past, and detracted from the future, but anything that has room for an increment, that can be turned into a greater magnitude, is by no means infinite.¹¹⁷ Furthermore, time is the measure of the spherical movements; and if the former is conceived to be beginningless, the latter must also have a claim to eternity. But those spherical movements are not uniform, there is a variety of ratios between them, while one sphere makes one revolution, another sphere may make three hundred and fifty-five revolutions. If the eternity hypothesis is correct, both spheres have made an infinite number of revolutions, yet sphere *B* must have certainly made 355 times as many revolutions as those of sphere *A*. Consequently one infinity would be greater than another infinity, which is absurd, because the infinite is greater than the greatest conceivable quantity.¹¹⁸ Hence temporal infinity is an impossibility. These arguments, it should be noted, are mentioned by Halevi¹¹⁹ among the proofs of the Mutakallimun for the theory of creation.

¹¹⁶ *Ibid.*, I, 59: . . . וכאשר מצאתי עצמי נמצא ידעתי כי ההויה עברה . . . על הזמן עד שהגיעה אלי ולולי שיש לי זמן תכלית לא היתה ההויה עוברת בה. See Guttman's *Die Religionsphilosophie des Saadia*, p. 40, note 3.

¹¹⁷ *Ibid.*, Part I, p. 74: והוא שכל יום חולף מהזמן לגלגל הוא תוספת: על מה שחלף וחסרון מן העתיד ומה שהוא סובל התוספת והחסרון יש תכלית לבחו ותכלית מחיבת החידוש.

¹¹⁸ *Ibid.*: וכאשר ראינו תנועות השמים מתחלפות עד שקצתם נערכים: על קצת על שלשים כפל ועל שלש מאות וחמשים וחמש ועל יותר מזה ידענו שכל אחד מהם יש לו תכלית.

¹¹⁹ See *Cosari*, Part V, ch. 18, First Axiom.

Bahya has the following to say about the infinite. He admits that number is infinite. There seems to be no end to the possibility of counting,¹²⁰ but actually everything is finite. Imagine a line AB drawn out *ad infinitum*, and take off a definite part AC

$\begin{array}{ccc} A & C & B \\ \hline \end{array}$

Now BC cannot be finite, for two finite lines make no infinite. But AB is of course greater than CB . Thus one infinite would exceed another infinite, which is absurd. Moreover, the very possibility of a part implies that the whole line must be finite, for a part bears a definite ratio to the whole, and is the unit of measurement. Indeed, the extensity of an object is that property of it by virtue of which it can be measured by a part. But the part can bear no ratio to the infinite. Consequently there can be no infinite extensity.¹²¹

After Bahya, a full century elapses, marking a blank in the history of the infinite, except perhaps for Gabirol's remarks that infinite, spatial or temporal, is due to formlessness, for that which has form must also be well defined in its limits—a purely Aristotelian position identifying the infinite with the indefinite.¹²² At last we come to Abraham

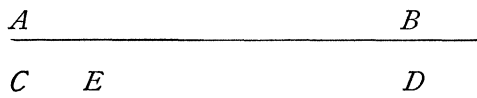
¹²⁰ See שער היחודי חובות הלבבות ; ch. 8 : אין תכלית למנין ; also ch. 5 : ואם נעלה במחשבתנו דבר שאין לו תכלית בפועל ונפריש ממנו קצתו יהיה השאר פחות ממה שהיה קודם מבלי ספק ואם יהיה השאר מאין תכלית יהיה דבר שאין לה תכלית גדול מדבר שאין לו תכלית והוא מה שא"א. This argument is mentioned in Spinoza's *Ethics*. See his note to Part I, prop. xv.

¹²¹ *Ibid.*, ch. 5 : מן הידוע כי כל מה שיש לו חלק יש לו כל כי אין הכל : ב"א כלל חלקיו ולא יתכן להיות חלק למה שאין לו תכלית.

¹²² *Fons Vitae*, IV, 6, p. 224 'Res autem non est finita nisi per suam formam quia res quae infinita est non habet formam qua fiat unum et differat ab alia ; et ideo essentia aeterna est infinita quae non habet formam.' Comp. V, 23, p. 300, and 29, p. 309.

Ibn Daud, who reiterates the Aristotelian position that only number, which has a potential existence, is infinite, but all actual things are finite. This thesis rests on the following four arguments, all except the first one being Aristotelian.

1. Let two lines AB and CD be drawn *ad infinitum*.



On CD mark off a finite segment CE . Let the line ED be superposed on AB so that point E coincides with point A . Now the question is, is ED equal to AB ? If ED equals AB , it will also equal CD , but how can a part be equal to the whole? If ED is less than AB , how can one infinity be smaller than another? And if ED is not infinite, how does ED plus CE , two finite lines, make an infinite line? This argument resembles Bahya's argument with one line.

2. There can be no infinite number of things, for a number is that which has been counted over, but infinity is that which cannot be counted over. Consequently an infinite number is a contradiction. Besides, a series has at least one limit, but in a beginningless and endless series all terms are intermediary. Consequently an absolutely infinite series is inconceivable.

3. An infinite body would not be in place, for that implies a containing body, and hence a larger magnitude than itself. But what is larger than the infinite? Here the reader may object that from the Aristotelian standpoint not all things are in space. The all-containing sphere is itself not contained.

4. An infinite body would not be at rest, for a body is

only at rest in its 'natural place', which an infinite body does not have. Nor would it be in motion, for a moving body leaves one place and occupies another place which it has not before occupied. But no place is free from the infinite. Hence an unlimited body is impossible.¹²³

A critical survey of these four arguments brings out a very important point. We find that the fourth argument is based on an absurd fiction of 'natural places'. The objection to the third has been given. It is the second argument that is truly valid, and defeats the first argument. It points out the absurdity of believing in a numerical or spatial quantity that is infinite. If quantity means anything at all, it is a well-defined relationship between the whole and a supposed part. The only difference between numerical and spatial quantity is that the one denotes a discrete nature and the other a continuous one. But whether it is ten discrete units or ten continuous inches, the relationship between the whole and the part is limited, nothing more and nothing less. Infinity, however, is that which has no limit, and hence cannot enter such relationship at all. Therefore an infinite quantity means nothing else than an infinite finitude, which is utterly meaningless. But if this is true, the fallacy of the first argument of Ibn Daud, and with it many more arguments that may possibly be fashioned after this model, becomes quite evident. If infinity has no quantitative relationships, of course nothing can be added to it or detracted from it—which means a change in those relationships; and the non-existence of infinity cannot be proved on that account. This point was noticed by Maimonides, and amplified by Moses Narboni.

¹²³ *Emunah Ramah*, pp. 15 ff.

In his exposition of the Kalam¹²⁴ Maimonides refers to some of the arguments adduced by that school against the infinite. Now Maimonides himself as an adherent of the Ptolemaic system of astronomy, and the creationistic theory, and as an opponent of the belief in a void, of course maintains absolute finitude in space as well as in time. Only he finds fault with the particular arguments on the basis of which the Mutakallimun negate infinity. They argue that if the world had no beginning in time, there would have elapsed up to this moment an infinite number of points and an infinite number of spherical revolutions and an infinite number of transient accidents. This whole process of fleeting moments and revolving spheres and transitory accidents still goes on, and a thousand years from to-day these infinities will be swelled by a certain number, and the infinity then will be greater than an infinite to-day.¹²⁵ Furthermore, if the eternity of the world is true, every celestial body has had an infinite number of revolutions. Now there is a definite ratio between these revolutions. While the terrestrial globe completes its circuit once a year, the lunar globe completes its circuit twelve times in a year. It makes no difference how long you allow these two spheres to revolve, the ratio will always remain 12 : 1. Now allow them to revolve *ad infinitum*, the numbers of their revolutions will be infinite ; but one infinity will be twelve times

¹²⁴ *Guide*, I, 74, seventh argument ; comp. *Cosari*, V, ch. 18, First Axiom.

¹²⁵ See also *Eṣ Ḥayyim*, ch. X: שאם לא כן ימצא דבר שאין תכלית לו יותר ממה שאין תכלית לו בשעור שנחסרו להויות באלף שנים בבא זה שאי אפשר: See also *Milḥamot*, p. 343: אחר סור זה ולרבר האחר נוספו שיחיה הזמן החולף בלתי בעל תכלית שאם היה אפשר זה לא יהיה רושם במה שיחדש מהתנועה בהוספת הזמן . . .

as much as the other, because the ratio subsisting between parts is also the ratio between their totalities, consequently infinity is impossible.¹²⁶ A more modern illustration than that of heavenly bodies may be found in dollars and cents. A dollar is to a cent as a hundred to one—a ratio which holds good for any number of these two coins; so that an infinite number of dollars will be a hundred times as much as an infinite of cents. You may invent many more such arguments from any system of weights and measurements, and you will get the same conclusion, contradicting the fundamental notion of the infinite, namely, that it is that greater than which is impossible.

But if we keep our previous conclusions clearly in mind, that the infinite, existent or non-existent, is no quantity, that it can enter into no quantitative relationships, it becomes evident first of all that a thousand years from to-day we will have no greater infinite, whether of temporal moments or spherical revolutions, than now; for the terms 'greater' and 'less' imply a quantitative whole, which infinity is not. And, secondly, it becomes evident that the ratio subsisting between parts falls off as soon as you enter the realm of the infinite, because the ratio is a quantitative relationship, and furthermore because the ratio between parts which is to hold good between their respective totalities is by no means similarly applicable to the infinite, which is not a quantitative totality. Thus as soon as you subject the infinite to mathematical calculations it slips as it were from your grasp, and what you are really dealing with is some big imaginary *finite* magnitude; but then, after you have drawn your conclusion, you exclaim

¹²⁶ Gersonides adduces the same argument in his *Milhamot*, p. 342. Similarly, see Spinoza, *l. c.*

triumphantly 'Eureka'. Maimonides therefore remarks very truly: 'The individual accidents that have passed into non-existence are counted and represented as though they were still in existence, and *as though they were things with a definite beginning; this imaginary number is then either increased or reduced.*' For it is evident that when you wish to add or detract you deal with a totality, and, as Aristotle remarked, the total and the infinite are mutually contradictory. The total is that beyond which there is nothing, and the infinite is that which admits of no beyond altogether. Infinite means endless, a being that is everywhere and whose existence, being immeasurable, cannot be expressed in any mathematical formula, and cannot be the basis of any mathematical equation.¹²⁷

The next man who grappled with this problem was Gersonides. I cannot allow myself, however, to omit two casual but characteristic remarks of two men living before him, Isaac Ibn Latif and Isaac Israeli. The former maintains¹²⁸ that the fact that our perception gives us the finite only, is not because reality is finite, but because our perceptive organs are unable to see the infinite.

¹²⁷ See Narboni, who expatiates on this idea which Maimonides puts very briefly and suggestively.

¹²⁸ חכמת המנין מציאותה מורגשת אלא שנמתחת : רב בעלים 63, section 63. והולכת עד לאין תכלית וכן חכמת השיעור הנקראת בלשון ערבי הנדסה גם היא מורגשת ונמתחת והולכת עד שנעלמת מן העין והנשאר מציאותה בשכל בלבד כדמיון מהלך שאין לה תכלית וכמציאות שני קוים שיש ביניהם בתחילת יציאתם שום מרחק על מה שיתרחקו ותחסר המרחק ויתקרבו האחד אל האחד ולא יתכן שיפגשו לעולם ואפילו יצאו לאין תכלית. . . . This last illustration Ibn Latif copied literally from the *Guide*, I, 73, prop. 10, where it is quoted from a certain *Book of Cones*, concerning which see Steinschneider, *Heb. Ueber.*, p. 169. It is also cited in the *Or Adonai*, p. 16 a.

That is why our mind does posit an infinite. Israeli, on the other hand, suggests¹²⁹ that though the human mind is capable of drawing the line and the surface and the solid *ad infinitum*, reality consists of finite and definitely-shaped objects. The former, Isaac Ibn Latif, was a Kabbalist, moving in a mysterious boundless atmosphere; the latter, Isaac Israeli, was a scientist busying himself with geometrical figures.

The Maimonidean suggestion that infinity does not denote any quantity, served as a starting-point for Gersonides. The latter, first of all, establishes that any quantity, whether numerical or spatial, is by its nature limited. This is a genuine Aristotelian conception. 'But', says Gersonides, 'we do not admit that the reason why matter, number, and magnitude are quantitatively finite is because they are actual, as the Philosopher holds, but because of the intrinsic nature of quantity, the proof of this being that number, even in the case of potential objects like time, must be limited nevertheless.'¹³⁰ Thus quantity is by its very definition finite. On the other hand, infinity is beyond any quantitative description. That is why the current definition of infinity as greater than the greatest conceivable body, is radically wrong. The difference between infinite and finite is not merely in *degree* but in *essence*. There is a wide unbridgeable chasm between these two natures. The infinite is irreducible to the finite, nor can the finite be enlarged to the infinite. Divide and subdivide the unlimited, if that is at all possible, and you

¹²⁹ See *Yesod Olam*, I, 2, p. 5 a: ידוע הוא שהשטח והקו והגוף הוא יוכל : אבל אין שום אמר מהם נמצא להמשך במחשבה אפילו עד אין חקר אבל אין שום אמר מהם נמצא בפועל אלא בעל תכלית ותמונה.

¹³⁰ *Milhamot*, pp. 336 ff.

are still within the realm of the unlimited.¹⁸¹ On the other hand, even if you were granted eternal life, and were to be engaged all your time in putting together particles of space, you would not step over the boundary of the finite. 'Just as a point will remain a point no matter how much you multiply it, because out of indivisibles you cannot get anything else than the indivisible; so magnitude will always remain magnitude, no matter how much you may multiply it; for it is *infinitely finite with all augmentation*.'¹⁸² The latter is a very pregnant saying: 'Magnitude is infinitely finite.' The infinite is not a product of an inconceivable number of finite spaces. It does not differ from the finite quantitatively, but qualitatively; it is altogether *sui generis*. What that essential quality is, is not quite clearly expressed. But the meaning seems to be this, namely, the removal in our thought of all quantitative determinations and limits. Focus your attention on the spatial fact itself, purely as a simultaneous co-existence without thinking of how far it is spatial, or on time purely as a successive flux, without thinking of the length of its duration; just as you may think of colour without regard to its space limits, and you have the notion of the infinite. Spatial infinity then might be defined as the representation

¹⁸¹ Thus he argues on p. 406, on the basis of this idea which can be expressed in the equation $\frac{\infty}{n} = \infty$, that if we divide infinite time into a finite number of times, we find ourselves in a baffling dilemma. The whole is naturally bigger than the part, but the part of an infinite is likewise infinite, how then can we conceive of two infinities, one greater than the other? Hence time is finite. Comp. also his argument from the 'Lunar Eclipse' on p. 342.

¹⁸² *Ibid.*, 345: תמיד לאין תכלית בעל תכלית (i.e. magnitude) כי הוא עם זאת ההוספה.

of the space-fact itself without regard to its quantitative aspect. This conception of the infinite is novel and interesting; it justifies the possibility of such a notion without involving oneself in numerous antinomies that arise out of a misunderstanding; and the emphasis that it lays on the idea that the infinite is not merely something greater than the greatest conceivable finite, marks an advance in history of the notion. The reader will note that Professor Fullerton recently urged exactly the same point, and on the basis of very much similar arguments.¹³³

But conception is one thing, and reality another. Such an abstract idea of the infinite is, like all abstractions, a purely mental fact. In reality, everything is limited and can be represented in a definite quantitative form; and

¹³³ See his *Conception of the Infinite*, ch. 2. I could hardly suspect Professor Fullerton of having read the *Milhamot*, but there is a very famous thinker in the history of modern philosophy who takes a similar view on the meaning of the infinite, and about whom such a suspicion might be ventured, I mean Baruch Spinoza. In Part I of his *Ethics* he lays down the proposition that substance absolutely infinite is indivisible; and anticipating some difficulty on the part of the reader to grasp the meaning of this paradoxical statement, he seeks to make it comprehensible (see note to prop. xv). But our study of Gersonides makes the meaning clear. The infinite is merely 'the representation of the space-fact itself without regard to its quantitative aspect', and is therefore indivisible. Only a definite quantity can be divided; spatiality as such is found in the same degree in a grain of sand and in the immeasurable ocean. The infinite designates space as a *quality* of matter and consequently suffers no diminution by any process of *quantitative* division. That this indeed is Spinoza's meaning is evident from his definition of eternity which is simply infinity in succession, namely, as 'existence itself in so far as it is conceived necessarily to follow solely from the definition of that which is eternal' and as distinguished from beginningless and endless continuity. Be it also remarked that from this standpoint the distinction between the infinite and the infinitesimal disappears, for the degree of largeness or smallness of matter plays no part in this conception of the infinite.

space is bounded with the bounds of the universe.¹⁸⁴ Yet there is one sense in which infinity can be said to be real, and that is *in process*. There is no end to the mental power of augmentation and diminution. There is no final term to a convergent series enlarging space by a certain unit, nor to a divergent series lessening space by a certain unit. Such a series may go on *ad infinitum*, though every term in that series is but a limited quantity, and gives us a sum total of a limited quantity. All this is because the human mind has acquired the ability to add and detract, and not having experienced anything that refuses addition or subtraction, it can conceive of no limit to that ability. But by addition and subtraction we can get nothing but finite results, so that this mental ability implies two apparently diametrically opposite things, namely, an infinite process with finite results. Indeed, the very exercise of this ability precludes any infinite result, for then the process would come to an end, inasmuch as nothing can be added to the infinite, and thus the process would no more be infinite. Yet the reader will ask, if infinite addition means anything at all, it means that there is no end to the process of adding, consequently there is no end to that which is added. But, as I have shown, if you analyse the term infinite addition, you find that it means that the additional process has no limit beyond which it cannot be carried, but an infinite result which cannot be augmented any more must set up a limit to the process. Hence the inference from infinity of process to infinity of state is

¹⁸⁴ *I. c.*, p. 339. See also p. 386 : ובכלל הנה אין כל מה שירמא האדם : צורק ולא ירמא האדם כל צורק אבל שם דברים צורקים לא יתבן שירמא אותם האדם כמו כלות העולם אל העדר המוחלט שאינו לא דקות ולא מלוי ומה שירמא ליה.

unjustifiable. That is why 'magnitude is infinitely finite'.

This explanation of Gersonides differs from the theory of potentiality as developed by Aristotle. He cautions¹⁸⁵ the reader not to understand by infinite divisibility or augmentation that a body harbours a possibility to be reduced into an infinitesimal or enlarged into an infinite, because that involves a misunderstanding of the infinite which really cannot be attained by means of the finite. All that is meant is, that a body, being extended, must be divisible; and inasmuch as it is a physical law that a body cannot be destroyed by division, every part must be further divisible. Similarly with augmentation, because any dimensional body has the quality of being enlarged. Thus two series set in, one convergent (1, 2, 3, 4, 5, &c.) and the other divergent ($1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \text{&c.}$). Both series run *ad infinitum*; and it is the condition of such a series, as has been shown, that no infinite term can be reached. Gersonides was more consistent than Aristotle, in making no discrimination between infinite divisibility and infinite augmentation.

Thus Gersonides's standpoint makes a genuine contribution to the history of this difficult problem. In completely severing the notion of the infinite from any quantitative relations, and in showing how infinity of process may, and indeed must, go hand in hand with finitude of state, Gersonides may still claim attention from modern thought. We will now pass to the next man, Hasdai Crescas.

The reader perhaps expects from Crescas a defence of the theory of the infinite; the expectation being based on

¹⁸⁵ *Ibid.*, 334.

two reasons: first, Crescas was the first in the history of Jewish thought to challenge Aristotelianism, and thus might have been led to renounce also the Aristotelian theory of the finitude of things; secondly, Crescas was, as we have seen, the first Jewish thinker to postulate pure space outside of and beyond the confines of the universe, thus space at least must be limitless. Well, the reader is not altogether wrong in his expectation, though not quite right. It is true that Crescas took issue with Aristotle on the subject of the infinite, and apparently he explicitly states that space is unlimited. 'It has been explained', he remarks in one place, 'that outside the world there must be either a full or a void, and that boundless dimensionality must exist. And even if it were non-existent, we would have to posit it, just as the geometrician makes use of such a concept in the definition of parallel lines and other fundamental terms.'¹³⁶ The latter comparison, however, already casts some suspicion on the author's meaning. The geometrician does not assume the infinite as a necessary fact, but as a hypothetical nature which must conform if real to the general laws and conditions of geometrical figures. It is only in this sense that we say two parallel lines are infinitely equidistant from one another. If now you make further investigation into the author's real opinion, you will find that Crescas at bottom adopted the view-point that was elaborated by Gersonides.

I said that Crescas took issue with Aristotle on the subject of the infinite. Indeed, he attacked all arguments of the Greek philosopher, as well as other arguments that were advanced in negating the idea later by Arabian scholastics. An exposition of this discussion in detail

¹³⁶ 'ה' אור, p. 16b.

that one infinity cannot be greater than another, the fact is that it cannot also equal another. Not only inequality, but also equality, is inapplicable to infinities. For even when we say that a thing equals, we have in our mind a whole quantum, in other words, a limited nature. Hence it is just as absurd to maintain that AB equals, as to maintain that it is greater than CB , for in either case we only *say* that we are dealing with the unlimited; in our mind, however, we have a definite measured amount which we try to compare with another equal or unequal amount. All mathematical considerations, all signs of equality and inequality, must be dropped entirely, if we really wish to conceive the endless. Else we are like the fabulous peacock that sought to escape its feet by flying.

Having this idea clearly in mind, we will find that the whole difficulty with this argument disappears. Let us take an example from time which is supposedly beginning-less. Up to now we have a series of moments infinite as to beginning, but limited by this present moment. A day passes by and a number of moments are added to the past. It does not mean, however, that the infinite has been 'increased', for this would suggest that we had a fixed calculable number of moments which we really did not have. We have a case of addition, but we cannot reduce it to a mathematical equation. What are you going to add it to? You are dealing here with unmathematical notions or metamathematical, if you will, and you have no right to

He thus overthrows Gersonides's argument against infinity from the infinite number of lunar eclipses, which not being greater, must be equal to, and coincident with, the infinite number of non-eclipses. According to Crescas one infinity can neither be greater nor equal to another, for it is altogether beyond the category of number. The whole passage is found verbatim in Abrabanel's *מפעלות אלהים*, IX, 7. See also above, end of ch. 2.

subject them to mathematical treatment. Similarly, you have drawn a line in space from this point *ad infinitum*, a yard further you have drawn a similar line. Both lines represent only an incomplete, so to speak, or unrealized infinite which must be endless as well as beginningless, leading from eternity to eternity. At any rate, all you have is a certain distance which might be added to the infinite line *B*. But to draw hastily a mathematical equation and to seek to get the net result, is to assume an imaginary finite line, or to have a wrong notion of what endlessness means.

The second argument is as follows :¹⁴⁰ If space is infinite we may select any point as a centre through which diameters run *ad infinitum*. The distance between any two diameters which form an angle at the centre becomes wider and wider until the intercepted arc would be infinite. Now the difficulty is twofold. First, if we imagine this infinite space to have a circular movement, how would the moving diameter cross this infinite intercepted arc? An infinity is just that which cannot be crossed over. Secondly, how can the arc be infinite when it is limited by the two diameters? and if it is not limited by them, the diameters must be finite. And if they are finite, the intercepted arc is naturally finite too.

Now, first, Crescas removes the objection from motion. It is inconceivable how an infinite body could move. To move means to leave an occupied place and to occupy an unoccupied place, but no place is free from the infinite. He now turns to the second difficulty. An intercepted arc

¹⁴⁰ אור ה', pp. 7 a, 16 b. This argument is in the main identical with Tabrizi's 'argument from scales', מופת הסולם. Cf. *l. c.*, p. 5 b. Comp. also Spinoza's *Ethics*, part I, prop. xv, note.

between two infinite diameters would eventually be infinite. But if it is infinite, how is it limited by the two diameters, and if it is unlimited by them, they must be finite. To this Crescas replies, an infinite line does not mean one that has infinite extent between its ends—a meaning which is of course contradictory and nonsensical. Similarly, it is absurd to look on this diameter for a point which will be an infinite distance from the centre; and inasmuch as the arc could be infinite only at such a point, it is evident that an infinite arc is impossible. What then do we mean by 'the infinite diameter'? Just this, that there is no limit to the possibility of extending the line, because space itself cannot be conceived to have limits; that it can be infinitely prolonged and nevertheless preserve its finite nature. This fact may at first seem strange, but it is no more strange, says Crescas, than the fact cited in the *Book of Cones*,¹⁴¹ that two lines starting at a distance from one another, and drawing nearer while they go on, never come in contact, even though you may prolong them *ad infinitum*. Infinity then denotes a process which may be perpetually carried on without breaking up the integral nature of the object, just as finitude denotes a limit which a certain process cannot surpass without destroying the peculiar nature of the object, as when we say that a body is only finitely divisible. Thus the diameter is infinite because it can endlessly be extended, though it always preserves its finiteness, though it never reaches a point which is at a boundless distance from the centre, and so never possibly intercepts an infinite arc. The reader will recall the pregnant saying, 'Magnitude is infinitely finite'. The key-note of this whole discussion is that there is an infinite process, which naturally implies finite results.

¹⁴¹ See above, note 128.

Thus there are two fundamental notions about the infinite which stand out very clearly from these two arguments. The first argument shows that infinity is in nowise reducible to terms of finitude and quantity, and vice versa. Hence the idea that we conceive the infinite by means of a successive synthesis of finites is erroneous. We may delve deep into the bottomless abyss, we may soar on our imagination to the dreary regions of pure space, we may make a life-long, or an eternity-long, successive synthesis, but we will still find ourselves much within the boundaries of the finite, simply because finite plus finite equals finite. It is not by widening limits, but by removing limits, by thinking away all quantitative determinations, that we are allowed a glimpse of the infinite.

The second argument obviates an objection from the reader, namely, if space can be endlessly enlarged, it must finally be endlessly large. The word 'finally' is not appropriate. Infinity denotes a process which is endless, consequently it has no final term. Hence there can be no infinite state or infinite result, because that would be a final term. The second argument then brings out the complementary idea that there is a logical harmony between infinity of process and finitude of results.

Thus we have seen how this conception as a whole was first faintly suggested by Maimonides, given prominence by Narboni, elaborated and crystallized by Gersonides, and finally clarified by Hasdai Crescas. It may, therefore, be justly called the view of infinity of mediaeval Jewish philosophy—a view that may claim even at the present day the serious attention of the student who is perplexed by the tangle of numerous contradictions and antinomies which this problem presents.

CONCLUSION.

A brief *résumé* of the chief points in the preceding discussion is now in order. I shall select the four central problems that have occupied our attention so far, and examine the solution offered by the mediaeval Jewish thinkers. These problems are: (1) the reality of empirical space, (2) the infinite divisibility of space, (3) the existence of absolute space, and (4) the infinity of space.

(1) In Jewish philosophy space is conceived as an objective reality. By 'reality' I understand the existence of a thing in the objective world independent of our perception. The mediaeval mind in general saw no problem in the reality of space. One might have disputed on how many angels could stand tip-toe on a pin-head, but that the pin-head exists with a certain magnitude of extension, no one entertained any doubt. It is only the modern mind, hypersophisticated, philosophically gone astray, that nervously asks whether this vast extension above and below and around us is not a mere illusion. Not only did the Jewish thinkers affirm the independent existence of space, but some even went so far as to take a geometric view of things and conceive the corporeal essence in terms of space. Matter, they maintained, is not merely that which takes up space, but it *is* space. All other characteristics that a certain object may possess are altogether unimportant for a pure conception of matter. A material object, according to these thinkers, may be defined as a limited magnitude of space that possesses certain qualities. Thus space and matter are synonymous terms. Other thinkers are less radical, and put space in the category of qualities. Corporeality means for them some mysterious substrate,

the conception of which requires no space determinatives. Yet in reality, all admit, space is inseparable from matter.

(2) But if unextended matter is an impossibility, it is evident that the Arabian atomic hypothesis, which reduces matter to ultimate non-magnitudinal parts, must be rejected. A non-magnitudinal part is in the first place impossible in itself, and secondly, how could it produce extension by combining with a similar part? A point is zero of extension, and you may add zeros *ad infinitum* without ever getting a number. Besides, the word 'combine' itself, if it is meant in a physical and not in a chemical sense, which is irrelevant in this connexion, implies a limit coming in contact with another limit, and a limit is a point before which there is a point which is no limit. In short, combination implies that that which combines is an aggregate of points, and consequently extended. Hence the idea that matter is composed of ultimate spaceless parts must be abandoned. The truth is, that no matter how much you may divide and subdivide a piece of matter, you will always get something that is further divisible. Of course, practically, you will eventually reach a *minimum sensibile*; theoretically, however, nothing prevents us from continuing with our process of division. Extension means 'alongsidedness of parts', and hence divisibility. Consequently, as long as you have matter you have divisibility. Therefore anything, however small and minute, can be divided *ad infinitum*. But here a dreadful gap opens up wide before us. If things are infinitely divisible, they must have an infinite number of parts, but how can a finite object contain an infinite number of parts? How can we move over even the smallest distance? And how could Achilles overtake the tortoise when the distance between

them is infinitely divisible, and each half of the distance that Achilles covers leaves another half between them, growing smaller and smaller to be sure, but never becoming zero? Indeed, one might ask how they can both begin to move, since the very first step, even that of the tortoise, involves a crossing of an infinite abyss? The fourth point, on the infinity of space, will give an answer to these questions also.

(3) So much for empirical space, or concrete extensity. This is undeniably real, as real as matter of which it is the distinguishing characteristic. But is there such a thing as pure space, mere dimensionality outside of and beyond the world of matter? Here opinions differed, the majority being against the existence of a void. In accepting the Aristotelian notion of space as 'the inner limit of the containing body', or a mere relation of contiguity between two objects, the Jewish thinkers had to endorse the exclusion of the possibility of pure space. For if by space, as distinguished from concrete extension, is meant merely contiguity, it is evident that where there are no bodies, there can be no space. This is precisely the Leibnizian position. Yet there is this critical remark to be made. Such a position might indeed explain the possibility of conceiving the vanishing of the space order, with the annihilation of the world of matter. But if this relationship of contiguity is to supplant the notion of space, by inheriting also its apodictic certainty; I mean, if the mind necessarily postulates such contiguity in connexion with matter; if an object cannot be conceived to exist outside of such relationship, the question may be asked, how is the universe as a whole conceivable without such relations? What, if pure space is denied, is con-

tiguous with the confines of the world? By what is matter limited? Indeed, such an objection, we have seen, was raised against the Aristotelian theory of the existence of a sphere which is all-containing and not contained. But the Jewish thinkers who negated the void would have flatly refused to confer 'apodictic certainty' on the relationship of contiguity. Some, it is true, were puzzled by the question: What is there beyond? And after they have proved by a series of arguments, to their own satisfaction, that space has limits and there is nothing beyond, they suddenly started at their own expression: Yes, but does not the word 'beyond' suggest a spatial background? The whole puzzle, however, was solved very truly by Abrabanel. The mind constantly receives spatial impressions from the external world, so that it has acquired a habit to consider things in spatial relations. Hence a solitary object that is shorn of these relations, is not easily conceivable, but it is not inconceivable. The human mind can transcend this habit and conceive of a finite totality which stands in no spatial relations with anything else.

(4) And so I come to the last point in our discussion. We saw in connexion with the idea of the void, that the finitude of space is held by the majority of Jewish thinkers. But infinite space presents a problem of its own. On the one hand many mathematical demonstrations might be made showing the impossibility of infinity; on the other hand, infinity seems to be a positive fact of experience. There can be no limit to the possibility of enlarging an object, just as we have seen that there can be no limit to the possibility of dividing a certain object. And if that is so, will not these two antithetical processes evolve two

bodies, one infinitely large, and the other infinitely small? Jewish philosophy has this to say on this serious difficulty. It is contradictory to speak of a body that is 'infinitely large' or 'infinitely small'. The terms 'large' and 'small' denote quantity, they present to our mind a definite *limited* magnitude; and infinity means *limitless*. Infinity, above all, must be absolutely distinguished from quantity; it is just by the removal of quantity that you conceive the infinite. And the fundamental error in the first Kantian autonomy is just this: that infinity is conceived as a successive synthesis of parts, whereas true infinity refuses being measured because it is just the reverse of measure, and excludes the notion of a part because it is indivisible as well as unaugmentable, being no definite magnitude, and is not obtained by a series of successive syntheses, because you may choose the greatest conceivable magnitude and multiply it by the greatest imaginable number, and what you will have will be a finite object as finite as a grain of sand and a blade of grass. Finite plus finite equals finite.

What then does infinity mean? It represents a process that may be carried endlessly without destroying the object; just as finitude represents such a process that will ultimately reach a limit, the crossing of which would spell injury to the object. It is in this sense that we say matter is infinitely augmentable, meaning that we can enlarge and further enlarge a given magnitude of matter *ad infinitum*, without ever producing an infinite magnitude, because that would mean the loss of matter which is by nature limited and circumscribed. Indeed, it is absurd to believe that such an infinite will eventually be reached, because then the process will cease, infinity being unaugmentable, and

the process will therefore be finite. Hence an infinite process presupposes finite results, and as one Jewish thinker cleverly remarked: Matter is infinitely finite. Similarly, infinite divisibility denotes that the process of division may be carried on theoretically *ad infinitum*, without bringing about the loss of the object. Yet this endless process never produces the infinitesimal, because that would involve the end of the process. But does not this mean, the reader will ask, that we could resolve a piece of matter into an infinite number of parts? No; first of all an infinite number is a contradiction of terms, and, secondly, if such an infinite number could possibly be attained the process of division would cease, but it is endless. Hence while each part becomes smaller and the number of parts greater, they cannot both overleap the boundaries of the finite. Thus Zeno's puzzles vanish like shadows in the light. We do not move over infinities, and Achilles can easily overtake the tortoise. What we have to bear in mind is only this, that infinity is a process, not a state.

Thus I have outlined briefly the Jewish standpoint in the problem of space, and I might conclude here perfectly well. Yet I should like to discuss one more point with the reader before we part. It is the Jewish empirical view versus the modern doctrine of the subjectivity of space. I fear that many a Kantian reader will leave this paper—if he looks at it at all—with a smile: Objectivity of space, Mediaevalism! Yet I believe that the phenomenalist theory has hindered rather than helped man in his desire to know his whereabouts, so as to adjust the interrelations in the best possible manner. Kant did not explain things, but transformed the world into a dreadful yawning abyss and called it Noumenon. He argued that we can mentally

annihilate and think away matter, but we cannot think away space, consequently space is a necessity of thought. But for myself, I cannot see how we can think away matter. Of course we can stop thinking at all, then we have thought away space, also ; but to think and not to think of things is absurd. When we think, of course we think something and about something. Objects of experience are the contents of our thought ; think away those objects, and thought becomes meaningless. And as for space being a necessity of the mind, Abrabanel, we have seen, explains it very clearly. It is a habit contracted by the mind under the pressure of constant spatial experience. Had the human mind been born in a spaceless universe, spacelessness would have become a necessity of thought. For what is consciousness if not the manifold impresses of external stimuli ? Hence the very idea that space is a necessity of thought proves that it is a necessity of reality. To deny this means to assume that the mind is some independent spiritual nature capable of engendering an order of existence. Of course, the infant undoubtedly has some dim sense of space, but this may have been because of the fact that the universal reality of space has developed in the human mind in the course of its evolution a spatial sense, because it helped the mind to adjust its relations to the external order ; and so this innate spatial sense is itself evidence for the reality of space. But I cannot take up this phase of the question here.

Thus I submit this Jewish empirical standpoint to the student of the problem of space, as a possible solution.